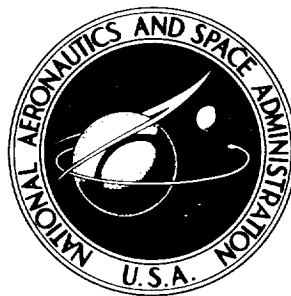


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**EFFECTS OF PEAK DECELERATION  
ON RANGE SENSITIVITY  
FOR A MODULATED-LIFT REENTRY  
AT SUPERCIRCULAR SPEEDS**

*by Charlie M. Jackson, Jr., and Roy V. Harris, Jr.;*  
*Langley Research Center,*  
*Langley Station, Hampton, Va.*

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### MODULATED-LIFT REENTRY AT SUPERCIRCULAR SPEEDS

By Charlie M. Jackson, Jr., and Roy V. Harris, Jr.

#### SUMMARY

Existing techniques were used to compute the flight-path parameters for a modulated-lift reentry maneuver which would define the undershoot boundary of the reentry corridor. Reentry conditions considered were velocities of 34,000, 36,000, and 38,000 feet per second and flight-path angles of  $-6^\circ$ ,  $-7^\circ$ ,  $-8^\circ$ ,  $-10^\circ$ , and  $-12^\circ$ . The effects of variations of peak deceleration on range and energy level of the vehicle at the end of the pull-up maneuver were investigated.

It was found that the longitudinal range and vehicle energy level at the end of pull-up were very sensitive to variations in peak deceleration when minimum peak deceleration was maintained. The maximum sensitivity occurred for reentry flight-path angles near  $-8^\circ$  for all reentry velocities considered. When the peak deceleration was increased by 0.5 percent of the minimum value, the sensitivity was reduced essentially to zero. Therefore, if deceleration is used as a trajectory control parameter along the undershoot boundary, this analysis indicates that a small concession in peak deceleration results in considerable reduction of the uncertainty of longitudinal range and vehicle energy level due to peak deceleration variations.

The effects of vehicle loading were considered for one reentry condition (that is, for a reentry flight-path angle of  $-8^\circ$  and a reentry velocity of 36,000 feet per second), and, although the magnitude of the range and vehicle energy level at the end of pull-up were considerably affected, the variation with peak deceleration was not. On the basis of this brief analysis, then, it would appear that vehicle loading has little, if any, effect on the range sensitivity to peak deceleration variations.

#### INTRODUCTION

The subject of entry into the earth's atmosphere from near parabolic orbits has received considerable attention with the increasing possibility of space travel. The feasibility of such a maneuver with respect to deceleration and heating has been theoretically demonstrated and the problems associated with maneuvering the vehicle to a predetermined landing point are being considered in some detail.

Reference 1 gives a general discussion of some of the more important reentry techniques for reducing deceleration, heating, and guidance requirements, along with an interesting study of the maneuvers associated with lateral and longitudinal range capability. Of the reentry techniques presented in reference 1, the modulation of lift from maximum lift coefficient to zero during the pull-up maneuver is most desirable from the standpoint of widening the reentry corridor at the undershoot boundary by allowing a steeper reentry for the design acceleration loading. Inasmuch as the exact position of the vehicle in the reentry corridor will in general not be known, the safest mode of operation, if it is close to the undershoot boundary, is to begin lift modulation when the design acceleration limit is reached. The details of this maneuver are discussed in references 2 and 3 and consist of maintaining a trajectory with maximum lift coefficient until the deceleration reaches the design value. Through pitch control the lift is then modulated toward zero, and the deceleration is maintained constant until the pull-up is complete. This pitch maneuver can be controlled either by the pilot or an automatic pitch control system. The question then arises concerning the sensitivity of the trajectory parameters to small variation in the magnitude of the deceleration force from the design value. The present report is concerned with these effects for undershoot boundary trajectories. The effects are analyzed with respect to longitudinal range achieved during pull-up and vehicle energy level at the end of pull-up.

#### SYMBOLS

The English system of units is used in this study. If conversion to metric units is desired, the following relationships apply:

$$1 \text{ U.S. foot} = 0.3048006 \text{ meter}$$

$$1 \text{ international statute mile} = 1,609.3440 \text{ meters}$$

$$C_D \quad \text{drag coefficient, } \frac{\text{Drag}}{\frac{1}{2}\rho V^2 S}$$

$$C_L \quad \text{lift coefficient, } \frac{\text{Lift}}{\frac{1}{2}\rho V^2 S}$$

$$\bar{E} \quad \text{energy parameter, } h + \frac{V^2}{2g}, \text{ ft}$$

$$g \quad \text{acceleration due to earth's gravity, ft/sec}^2$$

$$h \quad \text{altitude, ft}$$

$$m \quad \text{mass, slugs}$$

$$\dot{q} \quad \text{stagnation convective heating rate, Btu/ft}^2\text{-sec}$$

R	resultant aerodynamic force, lb
$R_e$	radius of earth, ft
$R/W$	magnitude of deceleration vector due to aerodynamic forces, g units
$\left(\frac{R}{W}\right)'$	peak deceleration due to aerodynamic forces, g units
r	vehicle nose radius, ft
S	reference area, sq ft
T	maximum stagnation radiation-equilibrium temperature, $^{\circ}R$
t	time, sec
V	velocity, ft/sec
W	vehicle weight, $mg_0$
$W/S$	vehicle loading, lb/sq ft
$\beta$	atmospheric density decay parameter, $ft^{-1}$
$\gamma$	flight-path angle measured positive up from local horizontal, deg
c	normalized deviation of peak deceleration from minimum, $\frac{\left(\frac{R}{W}\right)' - \left(\frac{R}{W}\right)'_{min}}{\left(\frac{R}{W}\right)'_{min}}$
$\rho$	atmospheric density, slugs/cu ft
$\psi$	longitudinal range angle measured from initiation of reentry, deg
Subscripts:	
E	conditions at start of reentry ( $h = 350,000$ ft)
min	minimum
o	conditions at sea level

A dot over a symbol denotes a derivative with respect to time.

## ANALYSIS

The reentry maneuver used throughout the present paper consists of a descent from an altitude of 350,000 feet at a constant pitch attitude ( $C_L = C_{L,max}$ ) until the magnitude of the resultant deceleration vector  $R/W$  reaches a specified value. (See fig. 1.) The lift coefficient is then modulated toward zero in order to maintain the specified value of  $R/W$ . When the flight path of the vehicle becomes horizontal ( $\gamma = 0^\circ$ ), the pull-up maneuver as referred to in this report is complete. For specific initial reentry conditions (velocity and flight-path angle at an altitude of 350,000 feet) there exists a minimum peak value of  $R/W$  with the condition that  $C_L \geq 0$ . The condition that  $C_L \geq 0$  is imposed from the standpoint of thermal protection weight. Because maximum heating occurs when the vehicle is at angle of attack, the lower surface and nose would require considerable thermal protection. If  $C_L \geq 0$ , then the upper surface of the vehicle does not experience the extreme thermal environment and, therefore, requires less thermal protection.

The equations of motion of reference 4 are modified and used in the present paper. The modification consists of the inclusion of an expression for the variation of the acceleration of gravity with altitude. The solution of these equations was obtained with a high-speed digital computer. For convenience the equations are listed as follows:

$$\frac{1}{g} \dot{V} = -\sin \gamma - \frac{\frac{1}{2}\rho V^2 C_D}{W/S} \quad (1)$$

$$\frac{V}{g} \dot{\gamma} = \frac{\frac{1}{2}\rho V^2 C_L}{W/S} - \cos \gamma \left[ 1 - \frac{V^2}{g(R_e + h)} \right] \quad (2)$$

$$\dot{h} = V \sin \gamma \quad (3)$$

$$\dot{\psi} = \frac{V \cos \gamma}{R_e + h} \quad (4)$$

where

$$\rho = \rho_0 e^{-\beta h} \quad (5)$$

$$g = g_0 \left( \frac{R_e}{R_e + h} \right)^2 \quad (6)$$



$$\frac{R}{W} = \frac{\frac{1}{2}\rho V^2}{W/S} \sqrt{C_L^2 + C_D^2} \quad (7)$$

In the present investigation the deviation of the peak deceleration  $(R/W)'$  from the minimum peak deceleration  $(R/W)'_{\min}$  will be expressed as

$$\epsilon \equiv \frac{\left(\frac{R}{W}\right)' - \left(\frac{R}{W}\right)'_{\min}}{\left(\frac{R}{W}\right)'_{\min}} \quad (8)$$

The following assumptions are made in the development of equations (1) to (6): (1) spherical earth, atmosphere, and gravitational field, (2) exponential variation of atmospheric density with altitude, and (3) nonrotating earth and atmosphere.

The variation of drag coefficient with lift coefficient (typical of asymmetrical lifting bodies designed for supercircular reentry) used for all calculations presented in this report is shown in figure 2. The values of the constants used in the equations of motion are given in the following table:

Acceleration of gravity at earth's surface, $g_0$ , ft/sec <sup>2</sup> . . . . .	32.174
Radius of earth, $R_e$ , ft $\times 10^6$ . . . . .	20.89
Atmospheric density decay parameter, $\beta$ , ft <sup>-1</sup> $\times 10^{-5}$ . . . . .	4.2553
Atmospheric density at sea level, $\rho_0$ , slug/cu ft . . . . .	0.0027

Except where otherwise noted, vehicle loading  $W/S$  equals 80.5 pounds per square foot.

According to the analysis of reference 5, the convective heating rate at the stagnation point can be expressed by the following equation:

$$\dot{q} = \frac{120 \times 10^{-12}}{\sqrt{r}} \left(\frac{\rho}{\rho_0}\right)^{1/2} V^{3.22} \quad (9)$$

where laminar flow is assumed. The radiation-equilibrium temperature at the stagnation point was calculated by using

$$T = \frac{4.198}{r^{1/8}} \left(\frac{\rho}{\rho_0}\right)^{1/8} V^{0.805} \quad (10)$$

which is obtained by equating the convective heat transfer (eq. (9)) to the heat radiated by the vehicle surface (assuming emissivity to be 0.8).

## RESULTS AND DISCUSSION

With the use of the techniques outlined in the section entitled "Analysis," the modulated-lift trajectories for the undershoot boundary of the reentry corridor were calculated for reentry velocities of 34,000, 36,000, and 38,000 feet per second and for flight-path angles of  $-6^\circ$ ,  $-7^\circ$ ,  $-8^\circ$ ,  $-10^\circ$ , and  $-12^\circ$ . The minimum values of peak deceleration  $(R/W)_{\min}'$  are presented in figure 3 for each of these reentry conditions. A practical value of reentry flight-path angle for the undershoot boundary appears to be about  $-8^\circ$  (based on a human tolerance of 10g as chosen in ref. 1).

In order to examine the effect of deceleration on the flight-path conditions at the termination of the pull-up maneuver, calculations were made for trajectories for  $\epsilon > 0$  (peak deceleration in excess of the allowable minimum for given reentry conditions). Typical results are shown in figure 4 which illustrates the variation of altitude  $h$  and lift coefficient  $C_L$  with range  $\psi$  for selected reentry initial conditions. The effect of  $\epsilon$  on range and altitude at the termination of the pull-up maneuver can be seen by comparing the terminal points of the trajectories (altitude variation with range) shown in figure 4. The variation of  $C_L$  with range (fig. 4) indicates that for specific reentry initial conditions the criteria that  $C_L \geq 0$  establishes the allowable minimum peak deceleration ( $\epsilon = 0$ ).

Figure 5 summarizes the effects on pull-up range of varying the magnitude of the peak deceleration from the minimum ( $\epsilon = 0$ ) for several reentry conditions. For some reentry angles a considerable effect on range is obtained with only a small change in  $\epsilon$ . For example, for a reentry angle of  $-8^\circ$  and a reentry velocity of 36,000 feet per second (fig. 5(b)), an increase of 0.05 percent over the minimum peak deceleration ( $\epsilon = 0$ ) results in a range-angle change of  $0.22^\circ$  or 15.25 international statute miles on the earth's surface. Figure 5 indicates that the effects of small variation of peak deceleration become more severe as  $\epsilon$  approaches zero (peak deceleration decreases).

These effects can be attributed to a complex interaction of the variation of dynamic pressure, lift coefficient, and flight-path angle as the vehicle descends. To analyze these variations, consider a typical case (fig. 4(b)). During the lift-modulation phase of the descent the dynamic pressure increases due to increasing atmospheric density, even though the velocity is decreasing. Concurrently, the lift coefficient must decrease in order to maintain a constant  $R/W$ . As  $C_L$  is reduced, the rate of pull-up  $\dot{\gamma}$  decreases and at some point (minimum  $C_L$  and maximum dynamic pressure) the rate of pull-up reaches a minimum value. At this point the vehicle is plunging into the atmosphere at peak dynamic pressure and with little or no pull-up (according to the value of minimum  $C_L$  attained) allowed because of acceleration limitations. The value of  $C_L$  (pull-up capability) at this point determines the duration of this plunge and the additional range accrued. A second-order effect is present. As the minimum  $C_L$  attained becomes smaller, the drag coefficient (decelerating force) decreases, and high dynamic pressure exists for a longer period of time; thus, the increase

of  $C_L$  ( $R/W$  must remain constant) is delayed, and the range required to complete the pull-up is increased. It is this second-order effect which causes the duration of the plunge, hence range, to increase in a nonlinear fashion as the minimum  $C_L$  attained decreases.

The slopes  $d\psi/d\epsilon$  of the curves shown in figure 5 are presented in figure 6 for values of  $\epsilon$  of 0 and 0.005. Figure 6(a) shows the effects of reentry conditions on range sensitivity  $d\psi/d\epsilon$  at minimum peak deceleration ( $\epsilon = 0$ ). Note that for the more practical undershoot boundary conditions ( $\gamma_E \geq -8^\circ$ ) the range sensitivity decreases with decreasing reentry flight-path angle. As the flight-path angle increases beyond  $-8^\circ$ , the effects of reentry conditions on range sensitivity are reduced.

Figure 6(b) illustrates the advantage of increasing the allowable peak deceleration by 0.5 percent of the minimum. This relatively small concession of increased deceleration reduces the range sensitivity to essentially zero. Spot calculations indicate that further increase in deceleration ( $\epsilon > 0.005$ ) produces little change in range sensitivity.

After the pull-up maneuver is complete, the energy of the vehicle is used to obtain additional range. This additional range usually amounts to greater than 90 percent of the total range, but, because there are many techniques of obtaining range (both longitudinal and lateral), a detailed analysis of the effects of peak deceleration on range for one specific range maneuver would yield little information of a general nature. The success of each range maneuver is primarily controlled by the energy at the end of pull-up; therefore, the effects of reentry conditions and deceleration on the energy level of the vehicle will be considered.

Figure 7 indicates the sensitivity of the energy parameter  $\left(\bar{E} = h + \frac{v^2}{2g}\right)$  with  $\epsilon$ .

The same trends occur for the energy parameter as for the pull-up range (compare figs. 5 and 7), except that the slopes of the curves are different in sign. Figure 7 therefore indicates that a slight increase in allowable peak deceleration ( $\epsilon$  increasing) results in a reduction of the energy-range sensitivity  $d\bar{E}/d\epsilon$ . Consequently, in the interest of reducing range uncertainties for a reentry along the undershoot boundary, it would be advantageous to compromise between range uncertainties and peak deceleration.

The effects of vehicle loading  $W/S$  were investigated for  $\gamma_E = -8^\circ$  and  $V_E = 36,000$  feet per second, and the results are presented in figure 8. Although the magnitude of the range angle and energy parameter is affected by the vehicle loading, the shape of the curves (fig. 8) and, therefore, the slopes remain essentially unaffected. It would appear, then, that vehicle loading has little, if any, effect on the range uncertainty due to peak deceleration inaccuracy.

As a matter of general interest, the effects of the reentry flight-path angle and velocity on the maximum stagnation-convective heating rate and radiation-equilibrium temperature for a sphere with a radius of 1 foot are presented in figures 9 and 10, respectively. Although these heating and temperature conditions seem extremely severe even for small reentry angles, it must be considered that

the maximum heating rate occurs over a short time and that the average heating rate is considerably less (about one-half for  $\gamma_E = -6^\circ$ ,  $V_E = 36,000$  feet per second). Also, the actual temperature experienced under transient conditions will be less than the radiation-equilibrium temperature.

#### CONCLUDING REMARKS

An analysis of the effects of peak deceleration on range sensitivity for a modulated-lift reentry maneuver which defines the undershoot boundary of the reentry corridor has been made for supercircular reentry velocities. The lift coefficient was modulated from maximum lift coefficient with the restraint that lift coefficient be equal to or greater than zero, and the resulting peak deceleration was obtained for several reentry conditions.

It was found that the longitudinal pull-up range and vehicle energy level at the end of the pull-up were very sensitive to variations in peak deceleration when minimum peak deceleration was maintained. The maximum sensitivity occurred for reentry flight-path angles near  $-8^\circ$  for all reentry velocities considered. When the peak deceleration was increased by 0.5 percent of the minimum value, the range sensitivity was reduced essentially to zero. Therefore, if deceleration is used as a trajectory control parameter along the undershoot boundary, this analysis indicates that a small increase in peak deceleration results in considerable reduction of the uncertainty of longitudinal pull-up range and vehicle energy level due to peak deceleration variations.

The effects of vehicle loading were considered for one reentry condition (that is, for a reentry flight-path angle of  $-8^\circ$  and a reentry velocity of 36,000 feet per second), and, although the magnitude of the range angle and vehicle energy level at the end of pull-up were considerably affected, the variation with peak deceleration was not. On the basis of this brief analysis, then, it would appear that vehicle loading has little, if any, effect on the range sensitivity due to the inaccuracy of peak deceleration.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., June 12, 1963.

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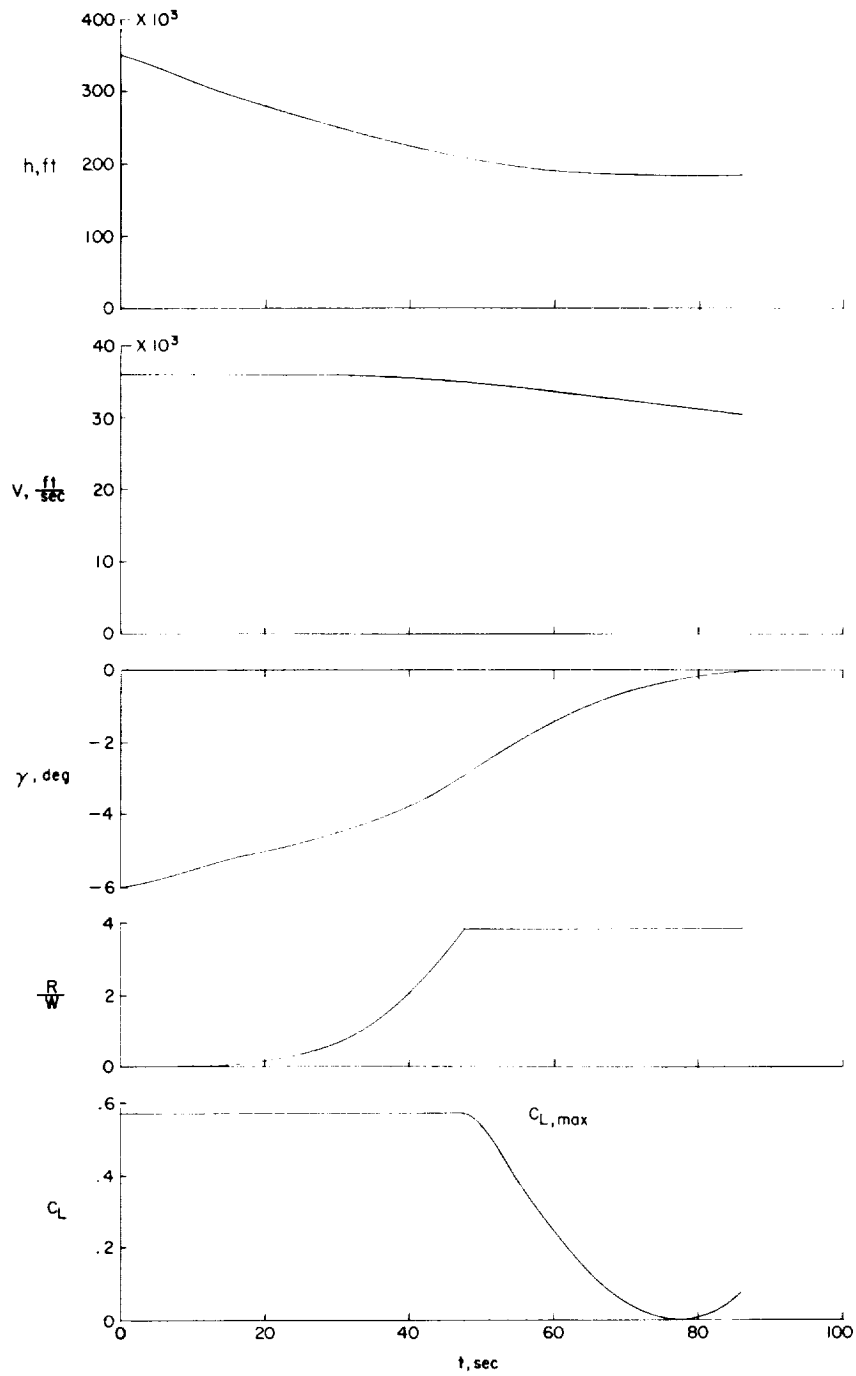


Figure 1.- Pull-up portion of typical trajectory for  $\epsilon = 0$ .

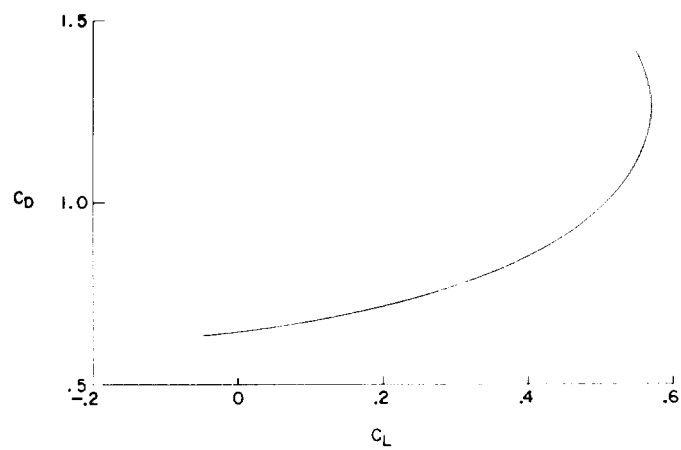


Figure 2.- Assumed variation of  $C_D$  with  $C_L$ .

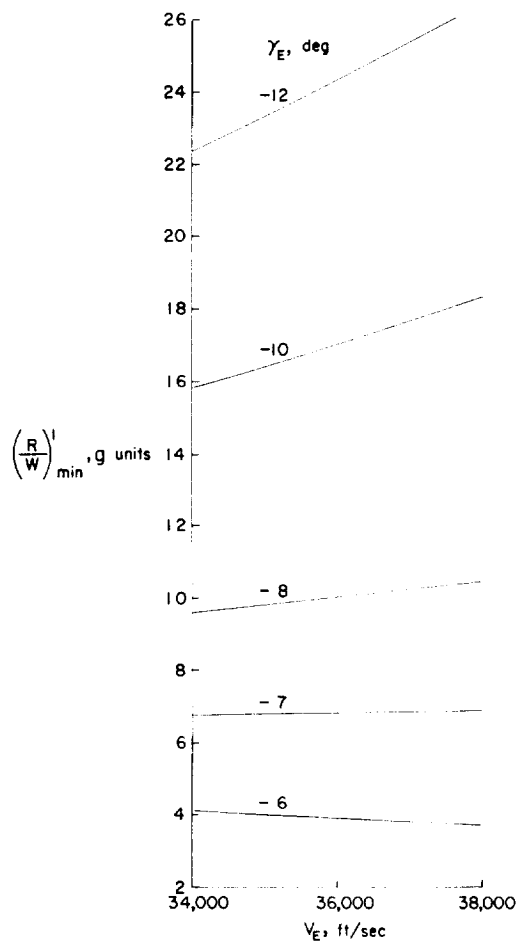
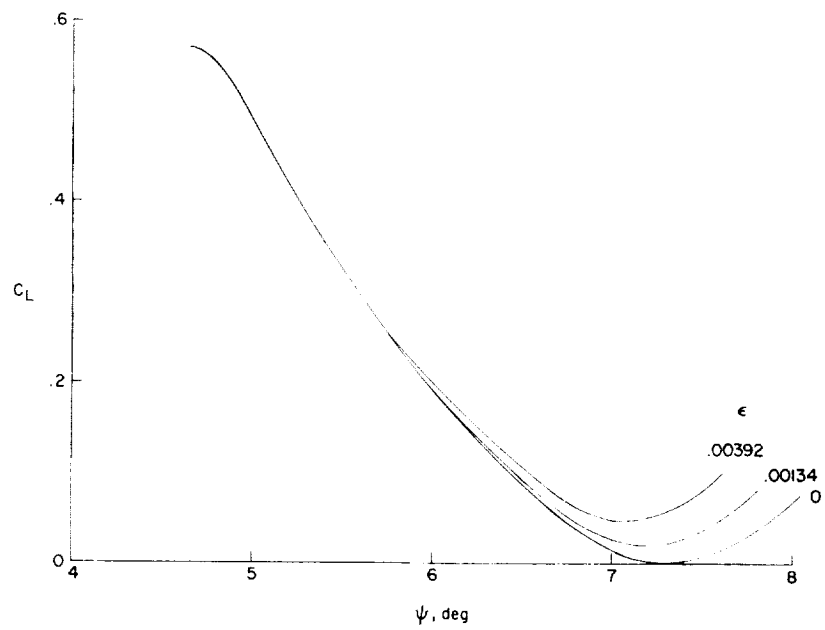
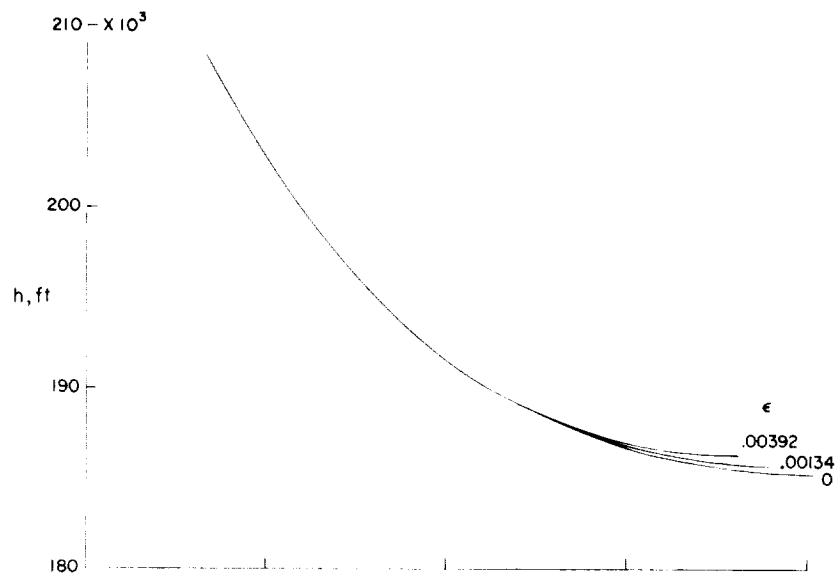


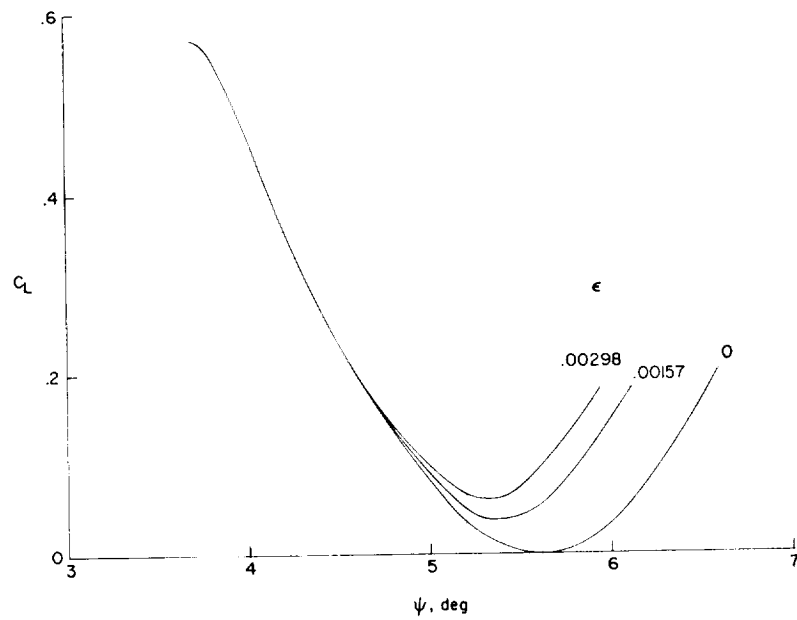
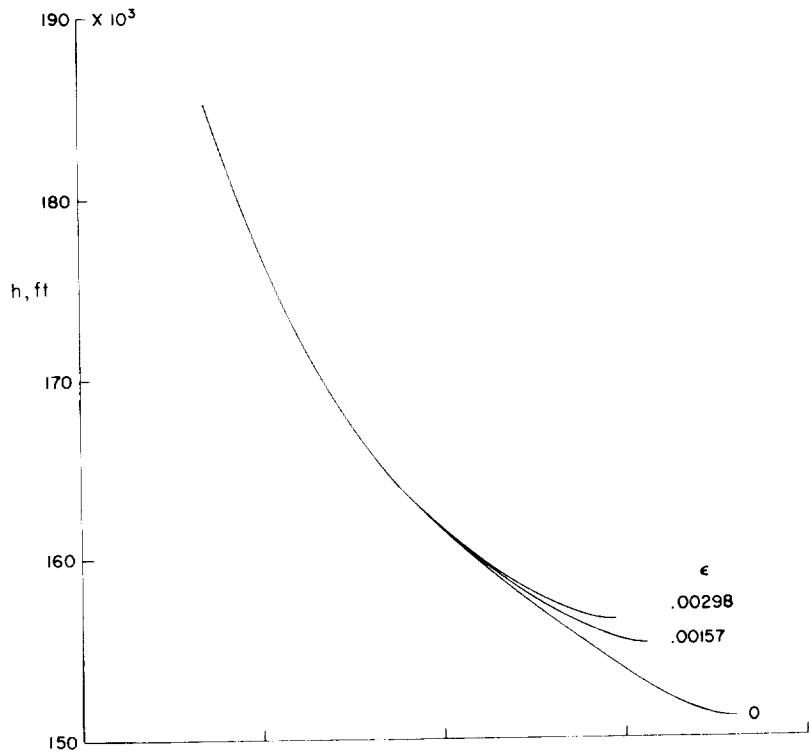
Figure 3.- Effects of entry velocity and flight-path angle on peak deceleration loading for pull-up maneuver.



(a)  $\gamma_E = -6^\circ$ .

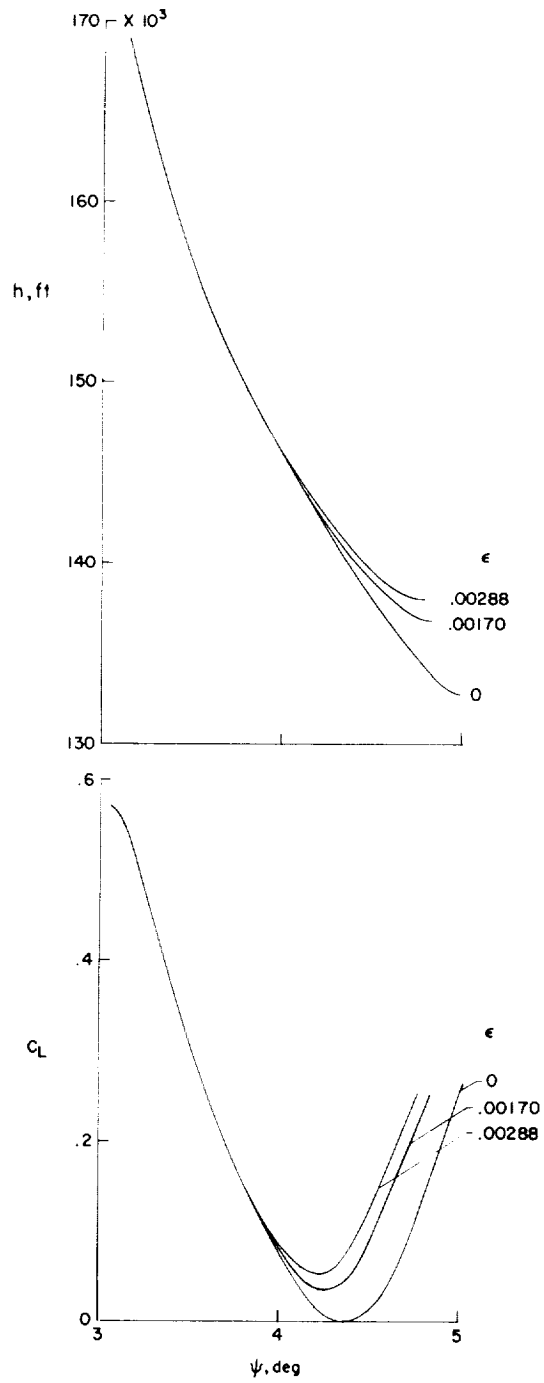
Figure 4.- Effects of reentry loading error on flight path and lift coefficient for pull-up maneuver.  $V_E = 36,000$  feet per second.





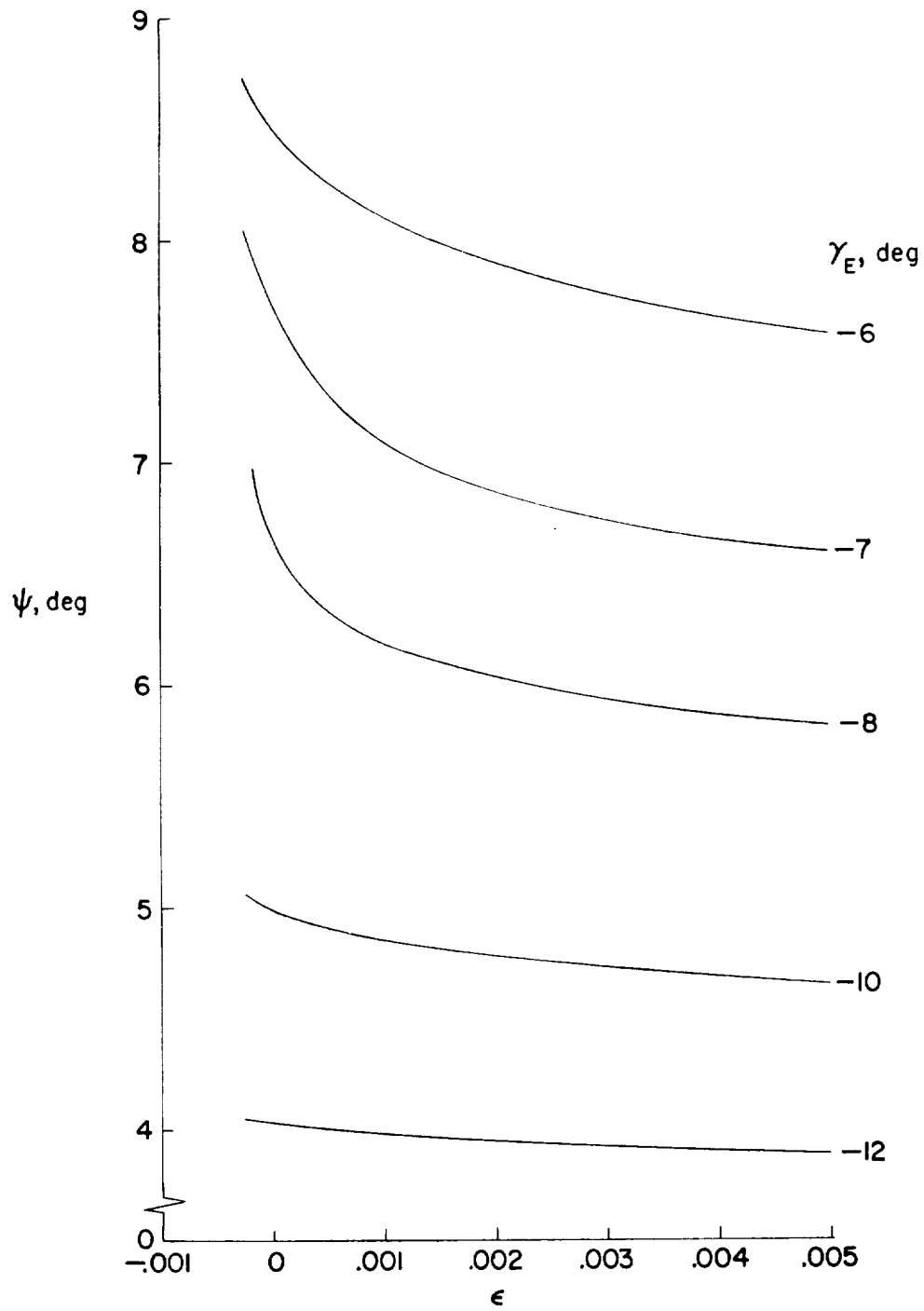
(b)  $\gamma_E = -8^\circ$ .

Figure 4.- Continued.



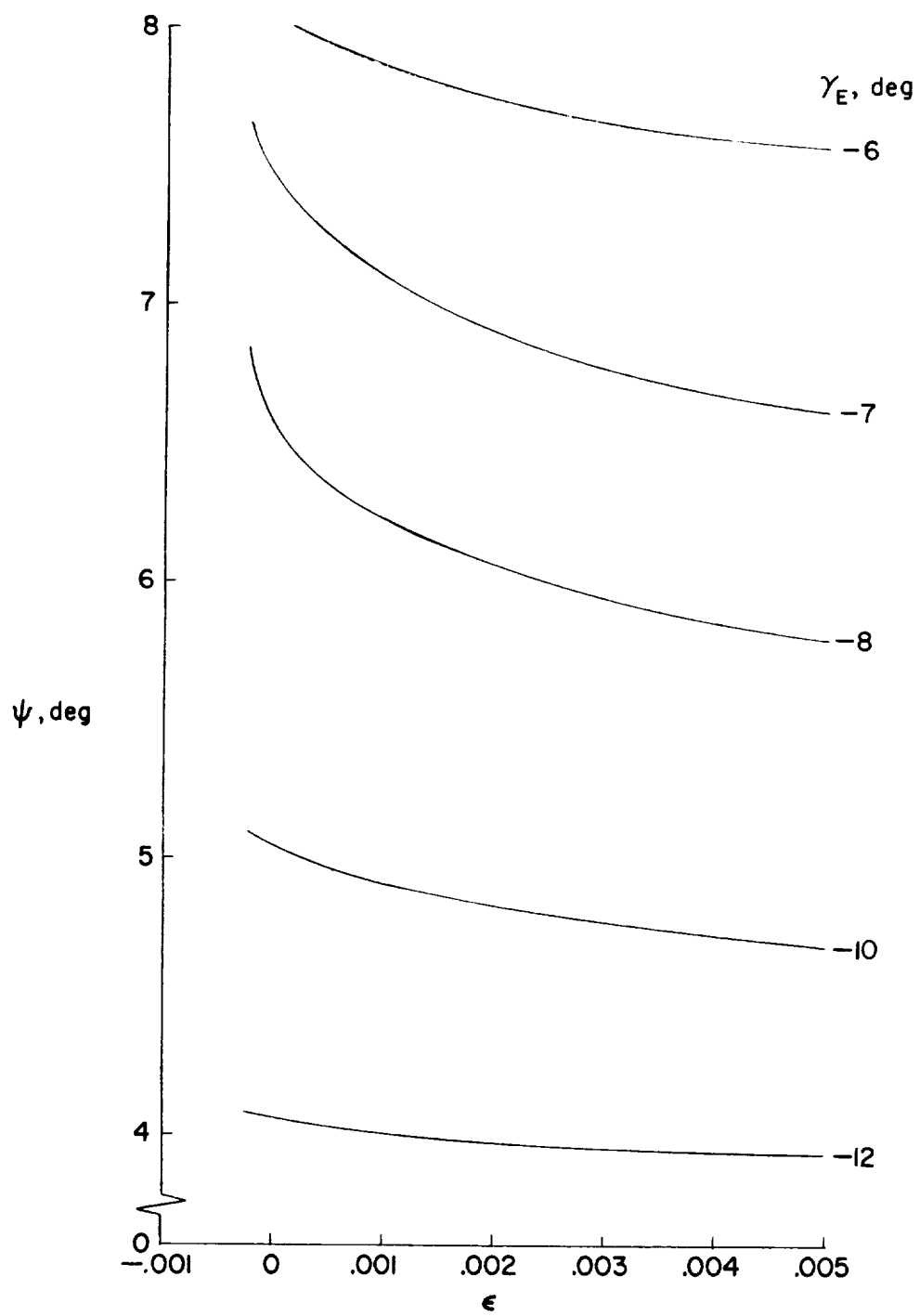
(c)  $\gamma_E = -10^\circ$ .

Figure 4.- Concluded.



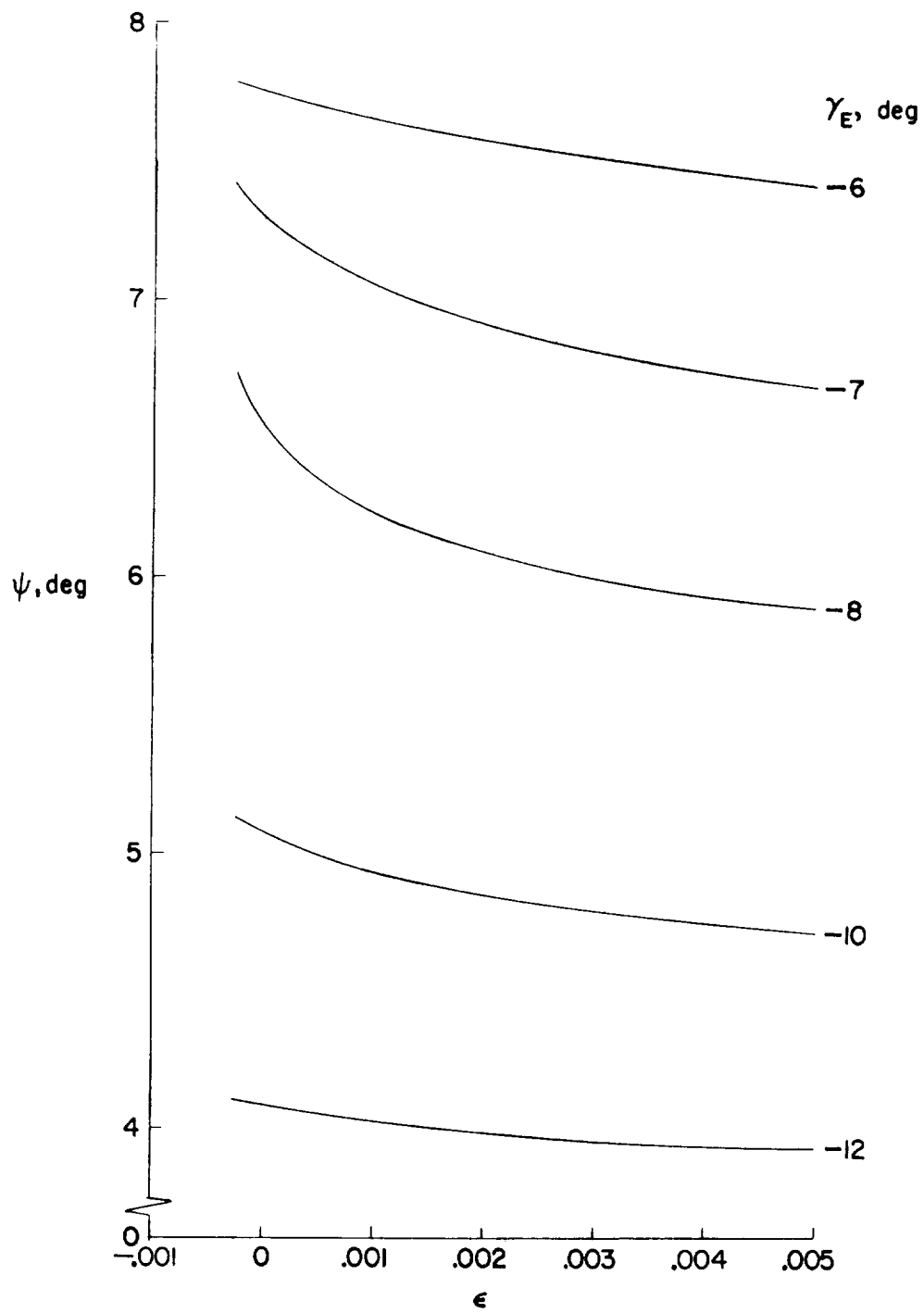
(a)  $V_E = 34,000$  feet per second.

Figure 5.- Variation of range  $\psi$  at end of pull-up with  $\epsilon$ .



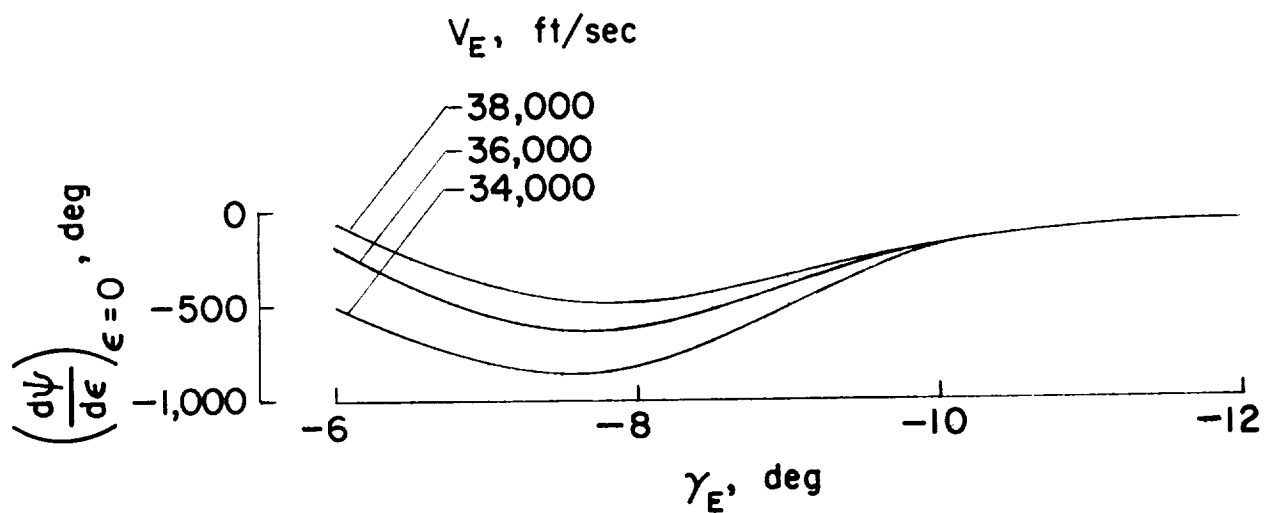
(b)  $V_E = 36,000$  feet per second.

Figure 5.- Continued.

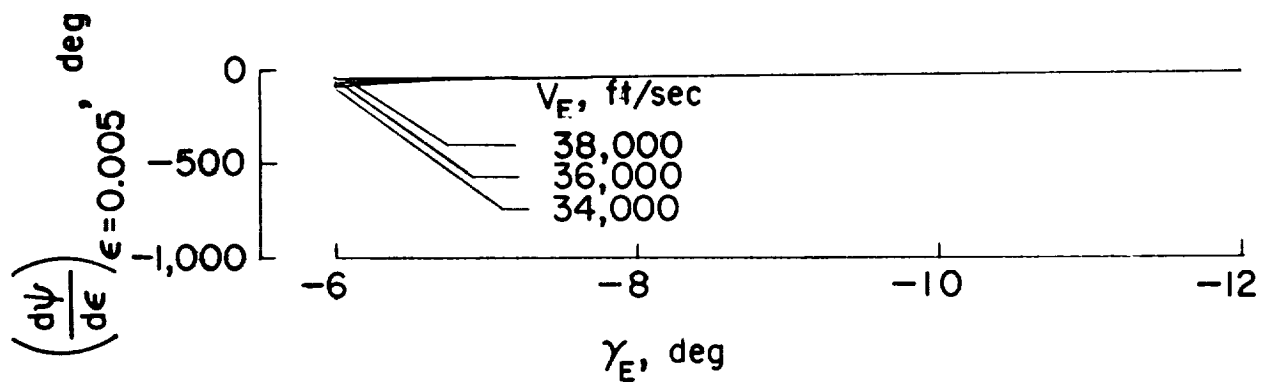


(c)  $V_E = 38,000$  feet per second.

Figure 5.- Concluded.

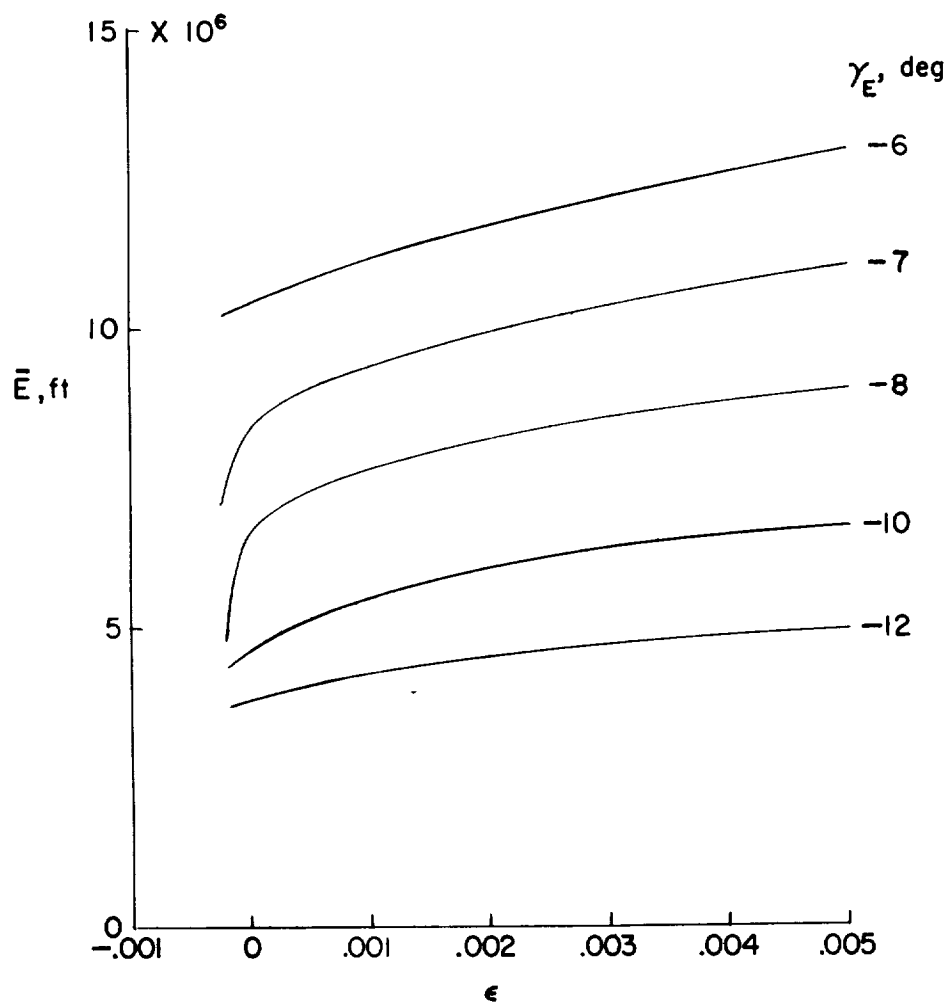


(a)  $\epsilon = 0$ .



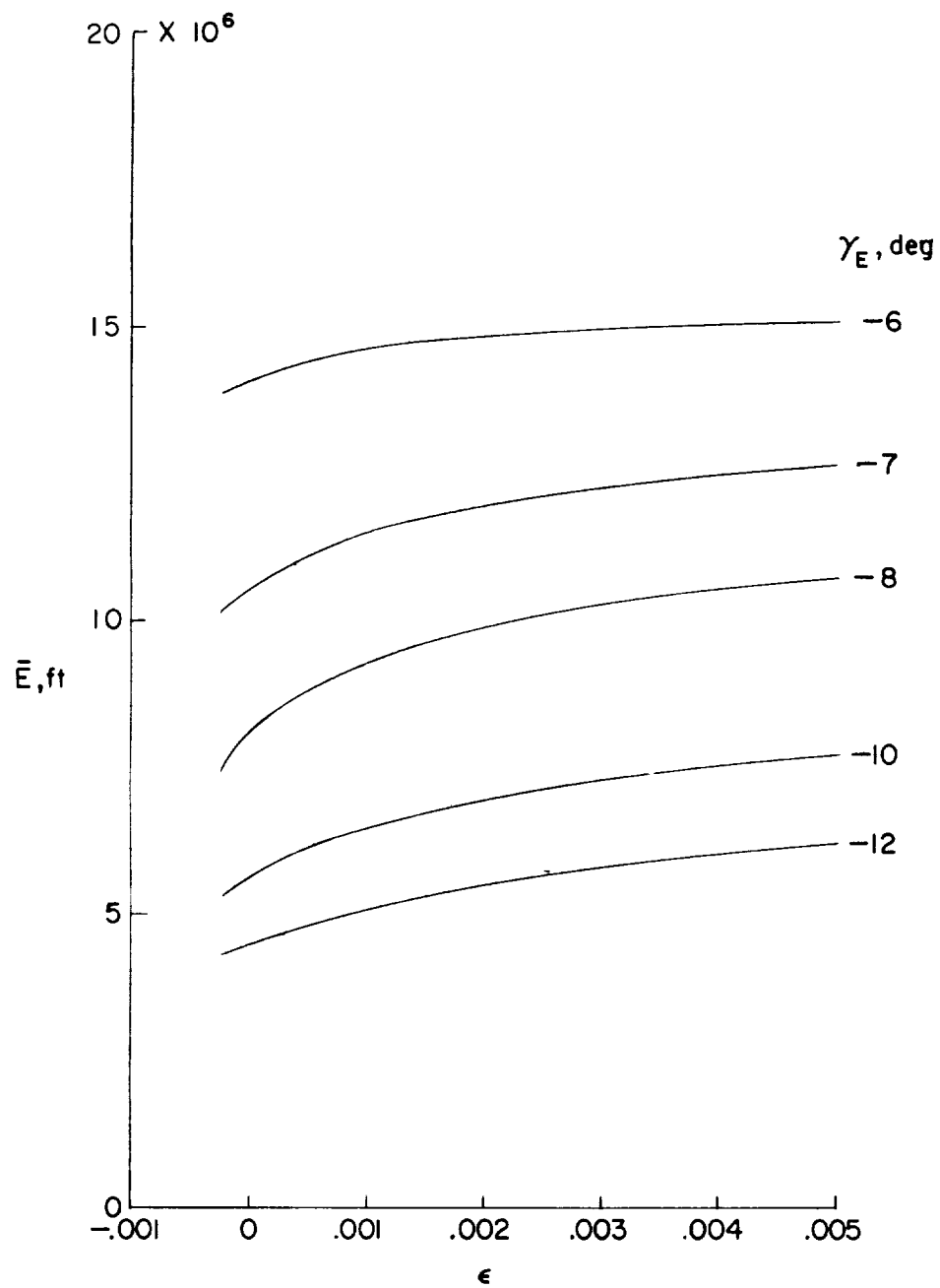
(b)  $\epsilon = 0.005$ .

Figure 6.- Effects of  $\gamma_E$  and  $V_E$  on range sensitivity.



(a)  $V_E = 34,000$  feet per second.

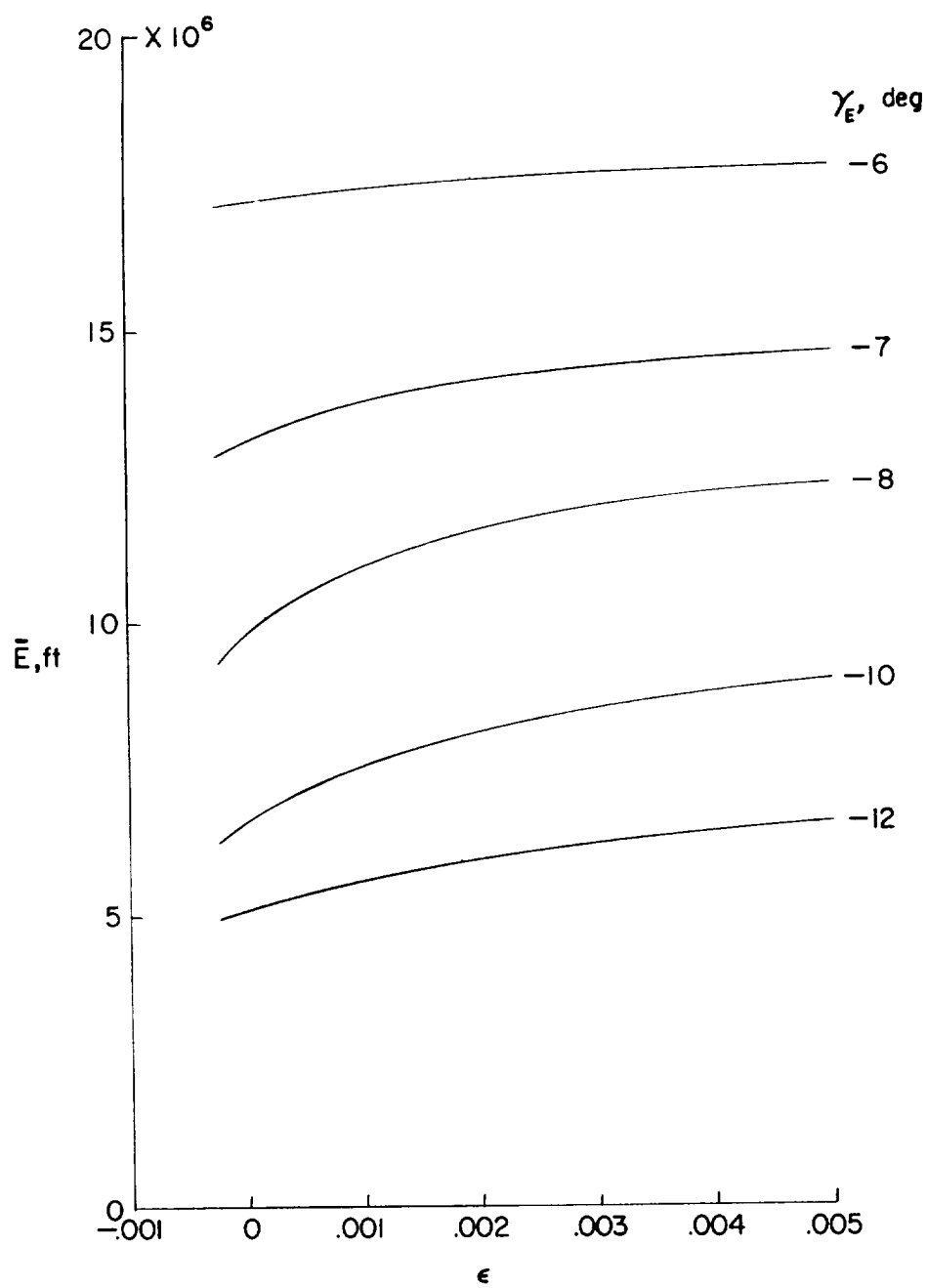
Figure 7.- Variation of energy parameter  $\bar{E}$  at end of pull-up with  $\epsilon$ .



(b)  $v_E = 36,000$  feet per second.

Figure 7.- Continued.





(c)  $V_E = 38,000$  feet per second.

Figure 7.- Concluded.

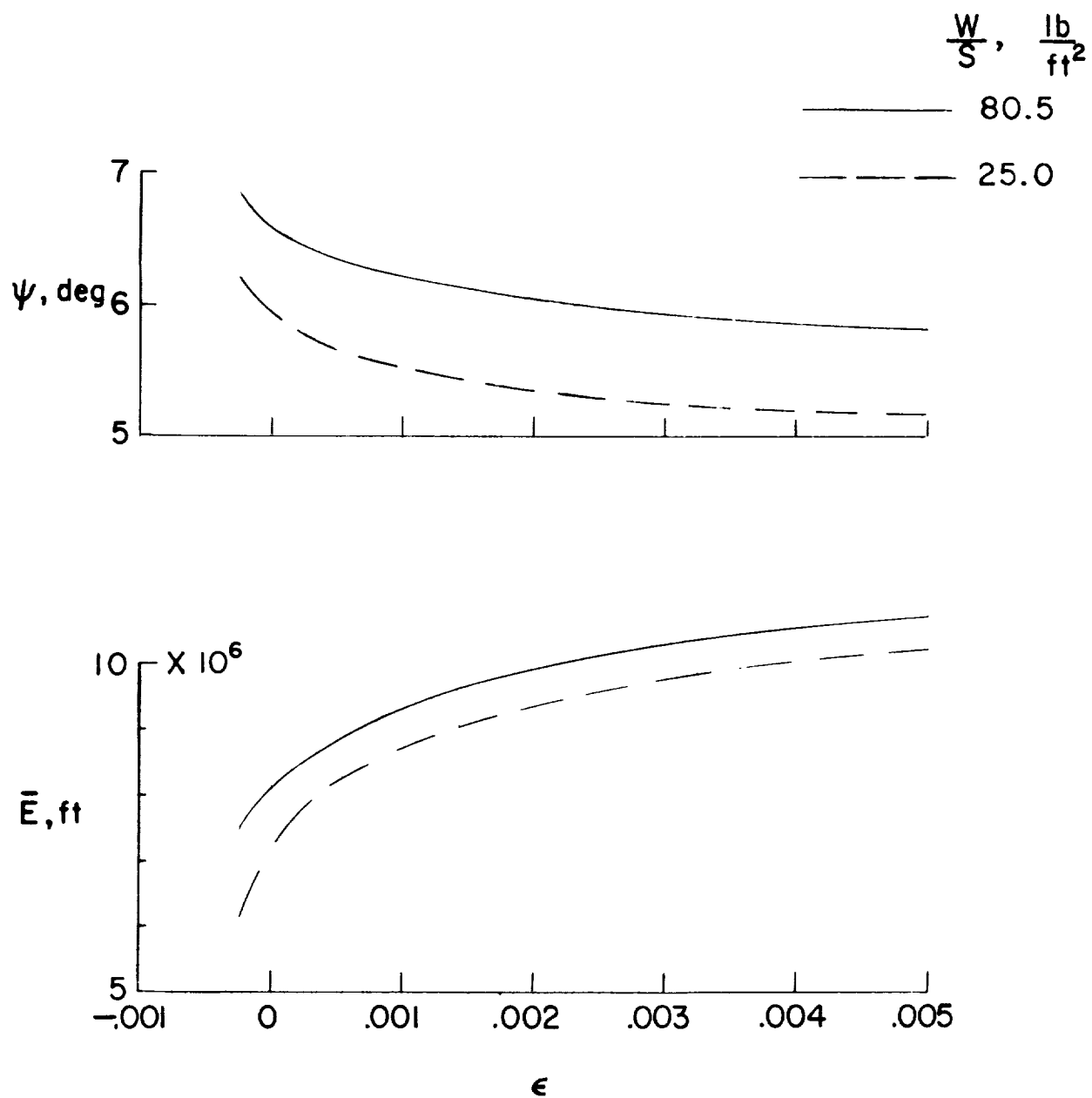


Figure 8.- Effects of reentry vehicle loading for  $\gamma_E = -8^\circ$  and  $V_E = 36,000$  feet per second.

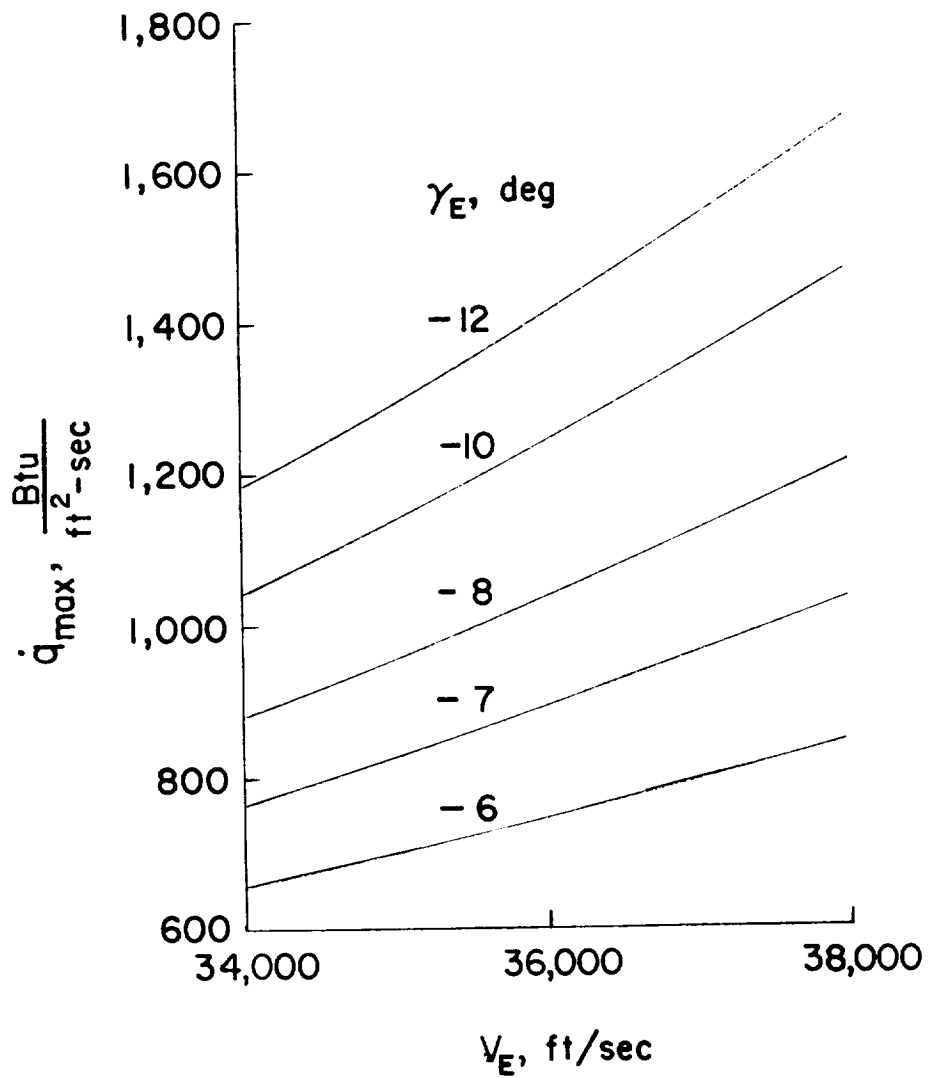


Figure 9.- Effects of entry velocity and flight-path angle on maximum stagnation heating rate of sphere with  $r = 1$  foot.

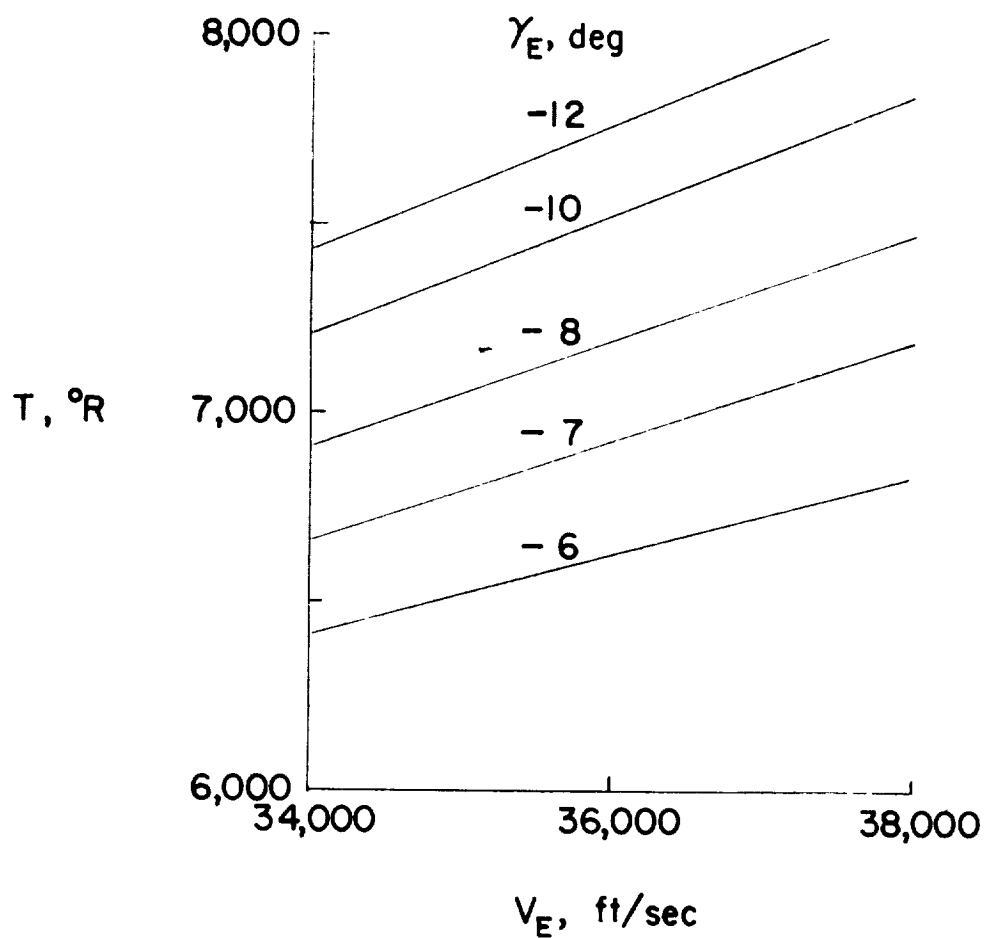


Figure 10.- Effects of entry velocity and flight-path angle on maximum stagnation radiation-equilibrium temperature of sphere with  $r = 1$  foot.



